Probability

* Probability = $\frac{\text{favourable no. of outcome}}{\text{Total no. of outcome}} = \frac{m}{n}$

* Complement of an event A is non-occurence of A. ie, A'

$P(A) = 1 - P(A')$

1) A bag has 9 tickets marked with numbers 1, 2, 3 .... 9. Two tickets are drawn at random from the bag. Find the probability that both the numbers drawn are (i) odd numbers (ii) even no.s

Sol: A : odd no.s are drawn = $\{1, 3, 5, 7, 9\}$

B : Even no.s are drawn = $\{2, 4, 6, 8\}$

(i) $P[\text{odd numbers}] = P[A] = \frac{m}{n}$

$= \frac{{}^5C_2}{{}^9C_2} = \frac{\frac{5\times4}{2\times1}}{\frac{9\times8}{2\times1}} = \frac{5\times\cancel{4}^{1}}{\cancel{2\times1}} \times \frac{\cancel{2\times1}}{9\times\cancel{8}_{2}}$

$= \frac{5}{18}$

(ii) $P[\text{Even numbers}] = P[B]$

$= \frac{m}{n}$

$= \frac{{}^4C_2}{{}^9C_2} = \frac{\frac{4\times3}{2\times1}}{\frac{9\times8}{2\times1}}$

$= \frac{\cancel{4\times3}^{1}}{\cancel{2\times1}^{1}} \times \frac{\cancel{2\times1}}{\cancel{9\times8}_{3.\ 2}} = \frac{1}{6}$

2) A bag contains 5 white, 4 red, and 2 green balls. Two balls are selected at random from the bag. Find the probability that the selected balls are (i) of the same colour (ii) of different colour

Soln:

A: Balls are of same colour

B: Balls are of different colour

(i) $P[\text{of same colour}] = P[\text{either 2 white OR 2 red OR 2 green}]$

$= P[\text{2 white}] + P[\text{2 red}] + P[\text{2 green}]$

$= \frac{{}^5C_2}{{}^{11}C_2} + \frac{{}^4C_2}{{}^{11}C_2} + \frac{{}^2C_2}{{}^{11}C_2}$

$= \frac{10 + 6 + 1}{55}$

$= \frac{17}{55}$

(ii) $P[\text{of different col}] = 1 - P[\text{of same colour}]$

$= 1 - \frac{17}{55}$

$= \frac{55-17}{55} = \frac{38}{55}$

3) Two cards are drawn at random from a pack of cards. Find the probability that (i) both are spades (ii) both are kings (iii) one is spade and other is heart (iv) belong to same suit (v) belong to different suits

Soln: A: selected cards are spades

B: Selected cards are kings

C: one spade and one heart cards are selected

D: Selected cards belong to same suit

E: Selected cards belong to different suits

52 cards → Spade, Club, Heart, Diamond (2, 3, 4 — 10, J, Q, K, A)

13 13 13 13

black Red

(i) $P(A) = \frac{^{13}C_2}{^{52}C_2} = \frac{1}{17}$

(ii) $P(B) = \frac{^{4}C_2}{^{52}C_2} = \frac{1}{221}$

(iii) $P(C) = \frac{^{13}C_1 \times {}^{13}C_1}{^{52}C_2} = \frac{13}{102}$

(iv) $P(D) = P[$either 2 spades, OR 2 clubs OR 2 heart OR 2 diamonds$]$

$= P[2 \text{ spades}] + P[2 \text{ clubs}] + P[2 \text{ hearts}] + P[2 \text{ diamonds}]$

$= \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2} + \frac{^{13}C_2}{^{52}C_2}$

$= \frac{4}{17}$.

(v) $P(E) = 1 - P[$cards belong to same suits$]$

$= 1 - \frac{4}{17}$

$= \frac{17-4}{17} = \frac{13}{17}$

4) A card is drawn randomly from a pack of playing cards. Find the probability that the drawn card is
(i) a spade or a king (ii) a king or a Queen.

[Addition theorem: Let A and B be two events with respective probabilities, P(A) and P(B). Then the probability of occurence of atleast one of these events is -
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$]

Soln: A: Drawn card is a king spade - 2, 3, 4 ... , 10, J, Q, K, A
B: Drawn card is a king. Spade, C, H, D
C: Drawn card is a queen

(i) $P[\text{spade or aking}] = P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$

$= \frac{13+4-1}{52} = \frac{16^{4}}{52_{13}} = \frac{4}{13}$

(ii) $P[\text{king or a queen}] = P(B \cup C)$

$= P(B) + P(C) - P(B \cap C)$

$= \frac{4}{52} + \frac{4}{52} - \frac{0}{52}$

$= \frac{4+4}{52} = \frac{8^{2}}{52_{13}} = \frac{2}{13}$

5. What is the probability that 4 cards are drawn at random from ~~the bag~~ a well-suffled pack of playing cards belong to different suits (one of each suit) ?

Soln:

P[cards of different / one of each suit] = P[one spade and 1 club, and 1 heart and 1 diamond]

$$= \frac{{}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1 \times {}^{13}C_1}{{}^{52}C_4}$$

$$= \frac{13 \times 13 \times 13 \times 13}{\frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1}}$$

$$= \frac{13 \times 13 \times 13 \times 13}{\frac{52 \times 51 \times 50 \times 49}{4 \times 3 \times 2 \times 1}} \times \frac{4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1}$$

$$= \frac{13 \times 13 \times 13}{17 \times 25 \times 49} = \frac{2197}{20825}$$