PART B

5. Write R script to perform the following using binomial distribution
   i. If n=4 and p=0.10, find P(x=3)
   ii. If n=12 and p=0.45, find P(5<=x<=7)

PROGRAM:

```
n1 <- 4
p1 <- 0.10
x1 <- 3
prob_x_3 <- dbinom(x1, size = n1, prob = p1)
cat("P(x=3) if n = 4 and p = 0.10 using binomial distribution:", prob_x_3, "\n")

n2 <- 12
p2 <- 0.45
x_lower <- 5
x_upper <- 7
prob_x_between_5_and_7 <- sum(dbinom(x_lower:x_upper, size = n2, prob = p2))
cat("P(5<=x<=7) if n = 12 and p = 0.45 using binomial distribution:", prob_x_between_5_and_7)
```

Output:

P(x=3) if n = 4 and p = 0.10 using binomial distribution: 0.0036

P(5<=x<=7) if n = 12 and p = 0.45 using binomial distribution: 0.583828

Solution:

5.

i If $n=4$ and $p=0.10$, find $P(X=3)$

Binomial Formula,

$$P(x) = {}^{n}C_{x} \cdot p^{x} \cdot q^{n-x}$$

$$= \frac{n!}{x!\,(n-x)!} \cdot p^{x} \cdot q^{x}$$

where, n = the number of trials (or the number being sampled)

x = the number of successes desired

p = the probability of getting a success in one trial.

$q = 1-p$ = the probability of getting a failure in one trial.

$n=4$, $p=0.10$, $q=1-p=0.90$, $x=3$

$$P(x) = {}^{4}C_{3} \cdot (0.10)^{3} \cdot (0.90)^{1}$$

$$= \frac{4!}{3!\,(4-3)!} * (0.10)^3 * (0.90)^1$$

$$= \frac{4!}{3! \times 1!} * (0.10)^3 * (0.90)^1$$

$$= \frac{4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2} \times 1 \times 1} * (0.10)^3 * (0.90)^1$$

$$= 4 * (0.001) * 0.9$$

$P(X=3) = 0.0036$

ii) If $n=12$, and $p=0.45$, find $P(5 <= x <= 7)$

This problem must be worked as the union of three problems:

a) $x=5$ b) $x=6$ c) $x=7$

a) $x=5$, $n=12$, $p=0.45$, $q=1-p=0.55$

$$P(x=5) = {}_{12}C_5 \cdot (0.45)^5 \cdot (0.55)^{(12-5)}$$

$$= \frac{12!}{5!\,(12-5)!} \cdot (0.45)^5 \cdot (0.55)^7$$

$$= \frac{\overset{3}{\cancel{12}} \times 11 \times \overset{2}{\cancel{10}} \times \overset{3}{\cancel{9}} \times \overset{4}{\cancel{8}} \times \cancel{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{(7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1)}} \cdot (0.45)^5 \cdot (0.55)^7$$

$$= (3 \times 11 \times 2 \times 3 \times 4) \cdot (0.45)^5 \cdot (0.55)^7$$

$$= 792 \times 0.0185 \times 0.0152$$

$$= 0.2227$$

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b) $P(x=6) = {}^{12}C_6 \cdot (0.45)^6 \cdot (0.55)^6$

$= \frac{12!}{6!(6!)} \cdot (0.45)^6 \cdot (0.55)^6$

$= \frac{\cancel{12}\times 11\times\cancel{10}\times\cancel{9}\times\cancel{8}\times 7\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}} \times (0.45)^6 \times (0.55)^6$

$= 2\times 11\times 2\times 3\times 7\times 0.0083\times 0.0277$

$= 924\times 0.0083\times 0.0277$

$= 0.2124$

c) $P(x=7) = {}^{12}C_7 \cdot (0.45)^7 \cdot (0.55)^5$

$= \frac{12!}{7!\times(12-7)!} \cdot (0.45)^7 \cdot (0.55)^5$

$= \boxed{\frac{12\times 11\times\cancel{10}\times\cancel{9}\times\cancel{8}\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times 1} \times (0.45)^7 \times (0.55)^5}$

$= (12\times 11\times 2\times 3) \cdot (0.45)^7 \cdot (0.55)^5$

$= 792 \times 0.0037\times 0.0503$

$= 0.1474$

Now sum the three probabilities

$P(5 <= x <= 7) = 0.2227 + 0.2124 + 0.1474$

$= 0.5826$

6. Perform the following using uniform distribution between 200 and 240.

   i. P(x>230)

   ii. P(205≤x≤220)

PROGRAM:

```
a <- 200

b <- 240

p_x_greater_230 <- 1 - punif(230, min = a, max = b)

p_205_x_220 <- punif(220, min = a, max = b) - punif(205, min = a, max = b)

cat("P(x>230) using uniform distribution between 200 and 240 is ", p_x_greater_230, "\n")

cat("P(205≤x≤220) using uniform distribution between 200 and 240 is ", p_205_x_220)
```

Output:

P(x>230) using uniform distribution between 200 and 240 is 0.25

P(205≤x≤220) using uniform distribution between 200 and 240 is 0.375

Solution:

6. Perform the following using uniform distribution between 200 and 240.

i $P(X > 230)$

$= P(230 < X < 240)$

$= \frac{x_2 - x_1}{b - a}$

$= \frac{240 - 230}{240 - 200}$

$= \frac{1\cancel{0}}{4\cancel{0}}$

$= 0.25$

(Write the first)
Probabilities in a Uniform distribution

$P(x) = \frac{x_2 - x_1}{b - a}$

where: $a \leq x_1 \leq x_2 \leq b$

ii $P(205 \leq X \leq 220)$

$= P(205 < X < 220)$

$= \frac{220 - 205}{240 - 200}$

$= \frac{\cancel{15}\,3}{\cancel{40}\,8} = \frac{3}{8}$

$= 0.375$