POORNAPRAJNA COLLEGE (AUTONOMOUS), UDUPI

NAAC Re-Accredited "A+" (3.27 CGPA)

(Promoted and Managed by Udupi Shree Adamaru Matha Education Council, Bengaluru)

SYLLABUS FOR

POSTGRADUATE PROGRAM (PG) OF

MATHEMATICS CURRICULUM FRAMEWORK

Course pattern and scheme of examination for PG Courses

FRAMED ACCORDING TO THE
STATE EDUCATION POLICY (SEP 2024)

M.Sc., Mathematics

I & II SEMESTERS

TO IMPLEMENT FROM THE ACADEMIC YEAR 2025-26

BOARD of Studies in M.Sc., Mathematics

POORNAPRAJNA COLLEGE (AUTONOMOUS),

UDUPI - 576101

A. Consolidated List of Courses:

The following shall be the Courses of study in the four semesters M.Sc. Mathematics Programme (CBCS- PG) from the academic year 2024-2025.

Hard Core Courses:

First Semester	Second Semester
1. MSMTCS 101 Algebra - I 2. MSMTCS102 Linear Algebra 3. MSMTCS 103 Real Analysis - I	4. MSMTCS 201 Algebra - II 5. MSMTCS 202 Real Analysis - II 6. MSMTCS 203 Topology

Soft Core Courses

First Semester	Second Semester
1. MSMTCS 104 Numerical Analysis 2. MSMTCS 105 Theory of Combinatorics 3. MSMTPS 101 Practical -I	4. MSMTCS 204 Ordinary Differential Equations 5. MSMTCS 205 Graph Theory 6. MSMTPS 201 Practical - II

Open Elective Courses

Second Semester
1. MSMTES 201 Discrete Mathematics and Applications.

Note:

1. All hard core courses are of 4 credits each and all are compulsory.
2. Practical courses are of 2 credits each and all are compulsory. For practical the student faculty ratio is 10:1. That is for every ten student one faculty to be allotted for effective implementation.
3. Soft core courses except practical courses are of 4 credits each. The soft core courses in the first two semesters are compulsory. In the third and fourth semesters student can choose any two soft core courses (other than practical courses) from the list of soft core courses offered in that semester.
4. Project work which is compulsory for every student, involves self study to be carried out by the student (on a research problem of current interest or on an advanced topic not covered in the syllabus) under the guidance of a supervisor.
5. Supervisor may be from the parent institution or from any other reputed institution/industry.
6. Project work shall be initiated in the third semester itself and the project report (dissertation) shall be submitted at the end of the fourth semester.
7. Project guidance shall be included in the teaching workload. Project guidance session for every two students will be considered as 1 hour per week. The student-to-faculty ratio for project supervision is 8:1.

B. Scheme of Instruction and Examination

First Semester

Course Code	Instruction Hours per week			Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
	Theory	PSS	Total					
MSMTCS 101	4	2	6	4	3	70	30	100
MSMTCS 102	4	2	6	4	3	70	30	100
MSMTCS 103	4	2	6	4	3	70	30	100
MSMTCS 104	4	2	6	4	3	70	30	100
MSMTCS 105	4	2	6	4	3	70	30	100
MSMTPS 101	4	-	4	2	3	35	15	50

Second Semester

Course Code	Instruction Hours per week			Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
	Theory	PSS	Total					
MSMTES 201	3	1	4	3	3	70	30	100
MSMTCS 201	4	2	6	4	3	70	30	100
MSMTCS 202	4	2	6	4	3	70	30	100
MSMTCS 203	4	2	6	4	3	70	30	100
MSMTCS 204	4	2	6	4	3	70	30	100
MSMTCS 205	4	2	6	4	3	70	30	100
MSMTPS 201	4	-	4	2	3	35	15	50

PSS: There shall be 2 hours of problem solving sessions (PSS) per week for each course having 4 credits and 1 hour problem solving session for an open elective. PSS are to be considered as teaching hours and in the PSS students are required to solve the problems in presence of the course instructor.

Scheme of Evaluation for Internal Assessment Marks:

1. Theory Course:
   Each Theory Course shall carry 30 marks for internal assessment based on two tests of 90 minutes duration each.
2. Project Work:
   Project Work shall carry 30 marks for internal assessment based on two presentations by the student before a panel of faculty members of the department.
3. Practical:
   Each Practical shall carry 15 marks for internal assessment based on two tests of 90 minutes duration each.

Pattern of Semester Examination:

1. Theory Paper:
   Each question paper for the theory course shall contain EIGHT questions out of which FIVE are to be answered. All questions carry equal marks.
2. Project Report:
   The evaluation of a project report is by two examiners as per the regulations.
3. Practical Exam:
   Each Practical exam question paper shall contain TWO questions on lab programmes which are to be executed.

C. Syllabi of Each Semester

I Semester

MSMTCS 101	Algebra- I	4 Credits (48 hours)

Course Objectives

1. **Develop a strong foundation in group theory:** Understand the fundamental concepts of groups, subgroups, cyclic groups, homomorphisms, isomorphisms, etc. Explore their applications in various mathematical contexts.
2. **Analyze symmetries and isometries:** Study the symmetries of plane figures and the properties of isometries to understand their mathematical significance.
3. **Explore advanced group theory concepts:** Delve into the class equation, p-groups, conjugation in symmetric groups, and the Sylow theorems. Learn their applications in different areas of mathematics.

4. **Understand the fundamentals of ring theory:** Grasp the basic definitions and properties of rings, polynomial rings, integral domains, fields, homomorphisms, ideals.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Understand and apply the laws of composition, recognize various types of groups (such as cyclic groups), and classify subgroups of given groups.
2. Demonstrate proficiency in understanding and using homomorphisms and isomorphisms, including the Correspondence Theorem to solve problems involving cosets and quotient groups.
3. Understand the concepts of isometries, finite groups of orthogonal operators, and apply these ideas to various mathematical problems.
4. Use group operations, Cayley's theorem, the class equation, and the Sylow theorems to analyze finite subgroups and conjugation in symmetric groups, and solve advanced problems in group theory.
5. Comprehend the definitions and properties of rings, polynomial rings, integral domains, fields, and their homomorphisms and ideals. Apply these concepts to solve problems involving quotient rings, adjoining elements, product rings, and maximal ideals.

Contents

Unit I – Groups (12 Hours)

Laws of Composition, Groups and Subgroups, Subgroups of the Additive Group of Integers, Cyclic groups, Homomorphisms, Isomorphisms, Equivalance Relation and Partitions, Cosets and Lagrange's Theorem, Modular Arithmetic, The Correspondence Theorem, Product Groups, Quotient Groups.

Unit II - Isometries and Operations on Groups (12 Hours)

Symmetry: Symmetry of plane figures, Isometries, Isometries of the plane, Finite groups of orthogonal operators on the plane.
Abstract Symmetry: Group Operations, The operation on Cosets, The counting Formula, Operations on subsets, Permutation Representations.

Unit III - Advanced Group Theory (12 Hours)

Finite subgroups of the Rotation Group, Cayley's theorem, The class equation, p-Groups, Conjugation in the symmetric group, Normalizers, The Sylow theorems and its Applications.

Unit IV – Ring Theory (12 Hours)

Definitions of rings, Polynomial Rings, integral domains, Fields and their basic properties, Homomorphisms and Ideals, Quotient Rings, Adjoining elements, Product rings, Fractions, Maximal Ideals.

References

[1] Michael Artin, *Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[2] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.

[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

[4] V. A. Herimath Algebra - Set Theory, Natural Numbers and Group Theory, Narosa, 2022.

[5] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.

[6] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.

[7] Thomas W. Hungerford, *Algebra*, Springer, 2004.

[8] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.

[9] Serge Lang, *Algebra,* 3rd Ed., Springer, 2005.

MSMTCS 102	Linear Algebra -I	4 Credits (48 hours)

Course Objectives

1. **Understand the fundamental concepts of vector spaces and linear transformations**: Grasp the principles of vector spaces, linear combinations, bases, dimension, linear transformations, null spaces, ranges, and matrix representations.
2. **Explore elementary matrices and determinants**: Learn about elementary matrix operations, matrix rank, inverses, and the properties and applications of determinants.
3. **Examine diagonalization and related matrix properties**: Study eigenvalues, eigenvectors, diagonalizability, The jordan form, Computation of invariant factors, and the Cayley-Hamilton theorem.
4. **Explore inner product spaces and associated linear operators**: Understand the Gram-Schmidt process, various linear operators and derive the spectral theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Understand and utilize vector space principles, linear transformations, matrix representations, null spaces, and ranges in solving mathematical problems.
2. Conduct elementary matrix operations, determine matrix rank and inverses, and calculate determinants of various orders with an understanding of their properties.
3. Find eigenvalues and eigenvectors, assess the diagonalizability of matrices, and apply related concepts such as the Cayley-Hamilton theorem.
4. Work with inner products, norms, the Gram-Schmidt orthogonalization process, orthonormal complements, and adjoint linear operators.
5. Understand and use normal and self-adjoint operators, unitary and orthogonal operators, orthogonal projections, the spectral theorem, singular value decomposition, pseudo-inverse, and the geometry of orthogonal operators in mathematical contexts.

Contents

Unit I (12 Hours)

Vector spaces: Recapitulation of Vector spaces, Linear combinations and system of linear equations, linear dependence and linear independence, Bases and dimension, maximal linearly independent subsets.

Linear transformations: Linear transformations, null spaces, and ranges, The matrix representation of a linear transformation, Composition of linear transformations and matrix multiplication, Invertibility and Isomorphisms, The change of coordinate matrix, Dual spaces.

Unit II (12 Hours)

Elementary matrices: Elementary matrix operations and elementary matrices, The rank of a matrix and matrix inverses, System of linear equations - theoretical and computational aspects.
Determinants: Determinants of order 2, Determinants of order *n*, Properties of determinants, A characterization of the determinant.

Unit III (12 Hours)

Diagonalization: Eigenvalues and eigenvectors, Diagonalizability, Invariant subspaces and Cyley-Hamilton theorem.
Cyclic subspaces and annihilators, Cyclic decomposition and rational form, The jordan form, Computation of invariant factors.

Unit IV (12 Hours)

Inner product spaces: Recapitulation of inner products and norms, The Gram-Schmidt orthogonalization process and Orthonormal complements.
The adjoint of a linear operator, Normal and self adjoint linear operators, Unitary and orthogonal operators and their matrices, Orthogonal projections and spectral theorem, Bilinear and Quadratic forms, The geometry of orthogonal operators.

References:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence – Linear Algebra, Prentice Hall of India, 4th Edition, 2014.
2. K. Hoffmann and R. Kunz – Linear Algebra, Prentice Hall of India, 2nd Edition, 2013.
3. S. Lang – Linear Algebra, Addison Wesley, London, 1970.
4. Michael Artin – Algebra, Prentice Hall of India, 2rd Edition, 2013.
5. S. Axler - Linear Algebra Done Right, Undergraduate Texts in Mathematics Springer, 4th Edition, 1997.
6. Larry Smith – Linear Algebra, Springer Verlag, 3rd Edition, 1998.
7. Gilbert Strang – Linear Algebra and its Applications, Cengage Learning, 4th Edition, 2006.
8. S. Kumaresan – Linear Algebra - A Geometric Approach, PHI, 2003.

MSMTCS 103	Real Analysis-I	4 Credits (48 hours)

Course Objectives

1. **Understand properties of number systems**: Learn about real and complex number systems, ordered sets, fields, and Euclidean spaces.
2. **Learn basic topology**: Explore metric spaces, open and closed sets, compact sets, and connected sets.
3. **Study numerical sequences and series**: Investigate convergent sequences, series, root and ratio tests, and power series.
4. **Explore continuity and differentiation**: Examine limits, continuous functions, derivatives, mean value theorems, L'Hospital's rule, and Taylor's theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Utilize properties of real and complex numbers, ordered sets, fields, and Euclidean spaces to solve mathematical problems.
2. Define metric spaces, identify open and closed sets, compact sets, perfect sets, and connected sets, and apply the Heine-Borel theorem.

3. Determine convergence of sequences and series using Cauchy sequences, root and ratio tests, and summation by parts.
4. Analyze function limits, identify and categorize discontinuities, and apply concepts of continuity, compactness, and connectedness.
5. Compute derivatives, apply mean value theorems, use L'Hospital's rule, and apply Taylor's theorems to real and vector-valued functions.

Contents

Unit I - The Real and Complex Number System: (12 Hours)

Introduction, Ordered sets, Fields, The real field, The extended real number system, The complex field, Euclidean spaces, Inequalities. Finite, Countable and Uncountable sets, Countability of Rational Numbers.

Unit – II Basic Topology: (12 Hours)

Metric spaces- Definitions, Examples and Basic properties, Open and Closed sets, Compact sets, Hein-Borel Theorem, Perfect sets – Uncountability of real numbers, Connected sets – Discussion in real line under usual metric.

Unit III - Numerical Sequences and Series: (12 Hours)

Convergent sequences, Subsequences, Cauchy sequences, Upper and lower limits, Some special sequences, Series, Series of non-negative terms, The root and ratio tests, Power series, Summation by parts, Absolute convergence.

Unit IV – Continuity and Differentiation: (12 Hours)

Limits of functions, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and limits at infinity.

The derivative of a real function, Mean value theorems, The continuity of derivatives, L’Hospital’s rule, Derivatives of higher order, Taylor’s theorems, Differentiation of vector valued functions.

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[4] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[5] R. R. Goldberg, *Methods of Real Analysis,* 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[6] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[7] Russel A. Gordon, *Real Analysis - A First Course*, 2nd Ed., Pearson, 2011.

MSMTCS 104	Numerical Analysis	4 Credits (48 hours)

Course Objectives

1. **Solving Transcendental and Polynomial Equations:** Understand methods for solving transcendental and polynomial equations, including convergence rates.
2. **Solving Linear Equations and Eigenvalue Problems**: Learn direct and iterative methods for solving systems of linear equations and eigenvalue problems.
3. **Interpolation and Approximation Techniques:** Study interpolation techniques and polynomial approximations, including least square approximations.
4. **Numerical Differentiation and Integration Methods:** Explore numerical differentiation and integration methods, including extrapolation and Newton-Cotes methods.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Use bisection, secant, Newton-Raphson, and other iteration methods, analyzing their convergence rates.
2. Apply direct methods (e.g., Gauss Elimination) and iterative methods (e.g., Jacobi, Gauss-Seidel), understanding convergence analysis and the power method.
3. Use Lagrange, Newton interpolations, finite differences, and Hermite interpolation for constructing interpolating polynomials and approximations.
4. Implement methods based on interpolation, finite differences, undetermined coefficients, and extrapolation for numerical differentiation.
5. Apply interpolation-based methods, Newton-Cotes formulas, and composite integration methods for accurate numerical integration.

Contents

Unit I - Transcendental and Polynomial Equations (12 Hours)

Introduction, The bisection method, Iteration methods based on first degree equation, Iteration methods based on second degree equation, Rate of convergence, Rate of convergence of Secant and Newton-Raphson method. Iteration methods - First order method, Second order method, Higher order methods. Polynomial equations, Descartes' Rule of Signs, The Birge-Vieta method, Ramanujan's method to find real /complex roots.

Unit II - System of Linear Equations and Eigen value problems (12 Hours)

Introduction, Direct Methods - Gauss Elimination Method, Gauss-Jordan Method, Triangularization Method, Cholesky Method. Iteration Methods - Jacobi Iteration method, Gauss- Seidel Iteration method, Convergence analysis, Eigen values and Eigen vectors. The Power Method.

Unit III - Interpolation and Approximation (12 Hours)

Introduction, Lagrange and Newton interpolations, Linear and Higher order interpolation, Finite difference operators, Interpolating polynomials using finite differences, Hermit interpolation, Approximations –Least Square Approximations.

Unit IV - Numerical Differentiation and Numerical Integration (12 Hours)

Numerical Differentiation: Introduction, Methods based on Interpolation, Methods based on finite differences, Methods based on undetermined coefficients, Extrapolation methods.

Numerical Integration: Methods based on Interpolation, Newton-Cotes methods, Composite Integration Methods.

References

[1] M. K. Jain, S. R. K. Iyengar, R. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Ed., New Age International, 2012.

[2] C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, Inc., 1999.

[3] A. Ralston and P. Rabinowitz, *A First Course in Numerical Analysis*, 2nd Ed., McGraw - Hill, New York, 1978.

[4] K. Atkinson, *Elementary Numerical Analysis*, 2nd Ed., John Wiley and Sons, Inc., 1994.

[5] P. Henrici, *Elements of Numerical Analysis*, John Wiley and Sons, Inc., New York, 1964.

MSMTCS 105	**Theory of Combinatorics**	**4 Credits (48 hours)**

Course Objectives

1. **Understand Fundamentals of Logic and Proofs**: Learn propositional logic, predicates, quantifiers, and various proof methods and strategies.
2. **Apply Counting Techniques**: Master basic counting principles, permutations, combinations, and advanced techniques such as inclusion-exclusion and derangements.
3. **Utilize Generating Functions and Recurrence Relations**: Study generating functions, their calculation techniques, and solve recurrence relations using various methods.
4. **Apply Group Theory to Counting Problems**: Understand group actions, the Orbit-Stabilizer Theorem, and apply Polya's Counting Principle and Cycle Index Polynomial.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Use propositional logic, predicates, quantifiers, and various proof methods to solve problems and construct rigorous arguments.
2. Apply basic counting principles, permutations, combinations, and advanced methods like inclusion-exclusion and derangements to solve complex counting problems.
3. Compute and apply generating functions, including exponential generating functions, and use summation operators for problem-solving.
4. Solve first-order and second-order linear recurrence relations with constant coefficients, including non-homogeneous cases, using generating functions.
5. Utilize group actions, the Orbit-Stabilizer Theorem, Polya's Counting Principle, and Cycle Index Polynomial to solve counting and enumeration problems.

Contents

Unit I – The Fundamentals of Logic and Proofs (12 Hours)

Propositional Logic, Applications of Propositional Logic, Propositional Equivalence, Predicates and Quantifiers, Nested Quantifiers, Rules of Inferences, Introduction to Proofs, Proof Methods and Strategy.

Unit II – Counting Techniques (12 Hours)

Counting: The Basics of Counting, Pigeon-hole Principle, Permutations and Combinations, Binomial Coefficients and identities, Generalized Permutations and Combinations.

Advanced Counting Techniques: Principle of Inclusion-Exclusion, Generalizations of the Principle, Derangements, Rook Polynomials.

Unit III – Generating Functions and Recurrence Relations (12 Hours)

Generating Functions: Introductory Example, Calculation Techniques, Partition of integers, Exponential Generating Function, The Summation operator.

Recurrence Relations: The First Order Linear Recurrence Relations, Second Order Linear Homogeneous Recurrence Relations with Constant Coefficients, Non- homogeneous Recurrence Relations, The method of Generating Functions.

Unit IV – Applications of Group Theory in Counting (12 Hours)

Group Action, Orbit Stabilizer Theorem and its applications to Polya's Counting Principle, The Cycle index Polynomial, Polya's Theorem Special Case and Applications, The Pattern Inventory, Polya's Theorem General Case and Polya's Inventory Problems.

References

[1] Kenneth H. Rosen, *Discrete Mathematics and its Applications*, 7th Ed., McGraw Hill, 2012.

[2] Ralph P. Grimaldi, *Discrete Combinatorial Mathematics*, 5th Ed., Pearson, 2006.

[3] D. I. A. Cohen, *Basic Techniques of Combinatorial Theory*, John Wiley and Sons, New York, 1978.

[4] Fred S. Roberts, Barry Tesman, *Applied Combinatorics*, 2nd Ed., CRC Press, 2009.

[5] G. E. Martin, *Counting: The Art of Enumerative Combinatorics*, UTM, Springer, 2001.

MSMTPS 101	**Practical -I**	**2 Credits (2 hours lab /week)**

Mathematics Practicals using Python Programming Language

Course Objectives

1. **Implement Basic Programming Constructs**: Develop programs to handle arrays, calculate factorials, and generate Fibonacci numbers using loops and conditionals.

2. **Perform Number System Conversions**: Write programs to convert between binary, octal, and decimal number systems using user-defined functions.
3. **Search, Sort, and Solve Equations**: Implement search and sorting algorithms, and apply methods to find roots of algebraic and transcendental equations.
4. **Apply Interpolation Techniques**: Develop programs to perform Lagrange, Newton Gregory, and Hermite interpolation methods for function approximation.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Write Python programs to find the largest/smallest element in an array and calculate factorials and Fibonacci numbers.
2. Convert numbers between binary, octal, and decimal formats using Python functions.
3. Use Python to perform linear and binary searches, and sort arrays in ascending and descending order.
4. Apply methods like Newton-Raphson, Secant, and Birge-Vieta to find real roots of equations.
5. Implement Lagrange, Newton Gregory, and Hermite interpolation methods to approximate functions.

List of Programs

1) Program to accept an array of numbers and print the largest/smallest among them (using 'if' – statement, 'elif'-statement and for loop).
2) Program to calculate factorial of a number and program to print Fibonacci numbers using 'for loop'.
3) Program to convert binary/octal number to decimal number and decimal number to binary/octal number using user defined functions.
4) Program to search an element in the array using linear and binary search.
5) Program to arrange a set of given integers in an ascending/descending order and print them.
6) Program to find roots of a quadratic equation.
7) Program to find a real root of a Algebraic/Transcendental equation using Newton Raphson Method/Chebyshev Method.
8) Program to find a real root of an Algebraic/Transcendental equation using Secant Method/Regula-Falsi Method.
9) Program to find a real root of a polynomial equation using Birge-Vieta Method.
10) Program to illustrate Lagrange interpolation.
11) Program to illustrate Newton Gregory Forward/Backward Difference interpolation methods.
12) Program to find the value of a function by using Hermite interpolation method.

II Semester

MSMTES 201	Discrete Mathematics and Applications	3 Credits (36 hours)

Prerequisite: Basic Mathematics up to XII/PU.

Course Objectives

1. Understand the fundamentals of number theory and their applications to cryptography.
2. Master various counting techniques and their applications in combinatorics.
3. Comprehend the properties and structures of relations, including partially ordered sets and Boolean algebras.
4. Develop problem-solving skills in number theory, combinatorics, and order relations.

Course Outcomes

1. Apply modular arithmetic and divisibility rules to solve complex problems in number theory.
2. Utilize counting techniques, such as permutations, combinations, and recurrence relations, in practical applications.
3. Analyze and interpret properties of relations, including extremal elements and lattice structures.
4. Implement cryptographic algorithms and understand their mathematical foundations.
5. Design and optimize circuits using Boolean algebra and finite Boolean algebras.

Contents

Unit I – Basics of Number Theory and Introduction to Cryptography:

Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Introduction to Cryptography. (12 Hours)

Unit II - Counting Techniques:

The Basics of Counting, The Pigeon-hole Principle, Permutations and Combinations, Binomial Coefficients and Identities, Generalized Permutations and Combinations, Recurrence Relations, Applications of Recurrence Relations, Solving Linear Recurrence Relations, Generating Functions. Principle of Inclusion-Exclusion, Applications of Inclusion-Exclusion. (12 Hours)

Unit III - Order Relations and Structures:

Product Sets and Partitions, Relations, Properties of Relations, Partially Ordered Sets, Extremal Elements of Partially Ordered Sets, Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Circuit Designs. (12 Hours)

References

[1] Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th Ed., Tata Mc-Graw-Hill, 2012.

[2] Bernard Kolman, Robert C. Busby, Sharon Cutler Ross, *Discrete Mathematical Structures*, 3rd Ed., Prentice Hall, 1996.

[3] Grimaldi R, *Discrete and Combinatorial Mathematics*, 5th Ed., Pearson, Addison Wesley, 2004.

MSMTCS 201	**Algebra - II**	**4 Credits (48 hours)**

Course Objectives

1. **Understand Factoring Techniques:** Study integer and polynomial factorization, including unique factorization domains, Euclidean domains, and irreducibility tests.
2. **Explore Field Extensions:** Learn about field extensions, including the degree of extension, algebraic vs. transcendental elements, and irreducible polynomials.
3. **Examine Advanced Field Concepts:** Understand field isomorphisms, splitting fields, primitive elements, algebraically closed fields, and finite fields.
4. **Apply Galois Theory:** Explore automorphisms, fixed fields, Galois extensions, and the main theorem of Galois theory, including its applications to polynomial equations.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Apply techniques for factoring integers and polynomials, including unique factorization and irreducibility tests.
2. Understand and find the degree of field extensions, and differentiate between algebraic and transcendental elements.
3. Determine isomorphisms between field extensions, and understand concepts like splitting fields and primitive elements.
4. Use Galois theory to analyze automorphisms, fixed fields, and solve cubic, quadratic, and quintic equations.
5. Explore the properties of finite fields and apply field extension concepts to solve related problems.

Contents

Unit I - Factoring:

Factoring Integers, Unique Factorization Domains, Euclidean domains, Content of polynomials, Primitive polynomials, Gauss lemma, Unique factorization in R[x], where R is a UFD, Factoring Integer Polynomials, Irreducibility test mod *p*, Eisenstein's criterion, Gauss primes.

(12 Hours)

Unit II – Fundamentals of Field Extensions

Definition and Examples, Characteristic of a Field, The Degree of Field Extension, Algebraic and Transcendental Elements, Finding the irreducible Polynomial, Ruler and compass constructions.

(12 Hours)

Unit III –Field Extensions and Finite Fields

Isomorphism of field extensions, Adjoining roots, Splitting fields, Primitive elements, Algebraically closed fields, The fundamental theorem of algebra, Finite fields and their properties.

(12 Hours)

Unit IV - Galois Theory:
Automorphisms and Fixed Fields, Galois Extensions, The Main Theorem of Galois Theory, Illustrations of the Main theorem, Cubic Equations, Quadratic Equations, Roots of Unity, Quintic Equations. (12 Hours)

References

[1] Michael Artin, *Algebra,* 2nd Ed., Prentice Hall of India, 2013.
[2] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.
[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.
[4] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.
[5] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.
[6] Thomas W. Hungerford, *Algebra*, Springer, 2004.
[7] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.
[8] Serge Lang, *Algebra,* 3rd Ed., Springer, 2005.

MSMTCS 202	Real Analysis - II	4 Credits (48 hours)

Course Objectives

1. **Understand Riemann-Stieltjes Integral**: Learn definitions, existence, properties, and applications of the Riemann-Stieltjes integral, including integration of vector-valued functions and rectifiable curves.
2. **Explore Sequences and Series of Functions**: Analyze point-wise and uniform convergence, Cauchy criterion for uniform convergence, and their implications for continuity, integration, and differentiation.
3. **Study Functions of Several Variables**: Master differentiation techniques, partial derivatives, directional derivatives, and key theorems such as the Contraction Principle, Inverse Function Theorem, and Implicit Function Theorem.
4. **Analyze Special functions and Function Spaces**: Understand special functions like exponential, trigonometric functions, gamma functions and their properties, and explore spaces of continuous functions, equicontinuous families, and approximation theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Compute and analyze Riemann-Stieltjes integrals, understanding their properties and applications to vector-valued functions and rectifiable curves.
2. Assess point-wise and uniform convergence of function sequences, and understand their effects on continuity, integration, and differentiation.
3. Use partial derivatives, directional derivatives, and apply the Inverse Function and Implicit Function Theorems to functions of multiple variables.
4. Analyze and work with special functions, including exponential, logarithmic, and trigonometric functions.

5. Work with spaces of continuous functions, equicontinuous families, and apply Weierstrass polynomial approximation to solve related problems.

Contents

Unit I - The Riemann-Stieltjes Integral:
Definition and existence of integrals, Properties of integral, Integration and differentiation, Integration of vector-valued functions, Rectifiable curves. (12 Hours)

Unit II - Sequences and Series of Functions:
Discussion of main problem, Point-wise Convergence and Uniform convergence, Cauchy criterion for Uniform Convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation. (12 Hours)

Unit III – Special Functions and Weierstrass Theorem
The spaces $C(X)$ and $B(X)$, Equicontinuous families of functions, Weierstrass Polynomial approximation theorem and its applications.
Some Special Functions: Power series, The exponential and logarithmic functions, The trigonometric functions, The Gamma function. (12 Hours)

Unit IV - Functions of Several Variables:
Differentiation, Partial Derivatives, Directional derivatives, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem. (12 Hours)

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.
[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.
[3] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.
[4] Serge Lang, *Analysis* I, Addison Wesley Publishing Company, 1968.
[5] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.
[6] R. R. Goldberg , *Methods of Real Analysis*, 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.
[7] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.
[8] Russel A. Gordon, *Real Analysis* - *A First Course*, 2nd Ed., Pearson, 2011.

MSMTCS 203	Topology	4 Credits (48 hours)

Course Objectives

1. **Understand Topological Spaces**: Learn the definition of topological spaces, including examples, elementary concepts, and open bases and subbases.
2. **Explore Compactness**: Study compact spaces, product spaces, and Tychonoff's theorem in the context of topology.
3. **Analyze Separation Axioms**: Investigate T1 and Hausdorff spaces, completely regular and normal spaces, and understand Urysohn's lemma, Tietze extension theorem, and Urysohn imbedding theorem.
4. **Examine Connectedness**: Learn about connected spaces, components, totally disconnected spaces, and locally connected spaces.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Identify and work with different types of topological spaces, open bases, and function algebras.
2. Demonstrate understanding of compact spaces and apply Tychonoff's theorem to problems involving product spaces.
3. Distinguish between various separation properties, and apply Urysohn's lemma, Tietze extension theorem, and Urysohn imbedding theorem.
4. Identify and analyze connected, totally disconnected, and locally connected spaces, and understand their components.
5. Use fundamental theorems and concepts to solve problems related to compactness, separation, and connectedness in topological spaces.

Contents

Unit I - Topological Spaces:

The definition and some examples, Open sets, Elementary concepts - Closed sets, Closure of a set, Kuratowski's Closure Axioms, Open bases and open subbases, Lindelof Theorem, Weak topologies, (12 Hours)

Unit II – Function algebras and Compact Spaces:

The function algebras *C(X, R*) and *C(X, C).* Compact Spaces, The Heine-Borel Theorem, Product spaces, Tychonoff 's theorem, The Generalized Heine-Borel Theorem, Compactness for metric spaces. (12 Hours)

Unit III - Separation:

T_1-Spaces and Hausdorff spaces, Completely regular spaces and Normal spaces, Urysohn's lemma and Tietze extension theorem, The Urysohn imbedding theorem. (12 Hours)

Unit IV - Connectedness:

Connected spaces, The components of a space, Totally disconnected spaces, Locally connected spaces. (12 Hours)

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

[2] J. R. Munkres, *Topology*, 2nd Ed., Pearson Education, Inc, 2000. (Add first Chapter).

[3] S. Willard, *General Topology*, Addison Wesley, New York, 1968.
[4] J. Dugundji, *Topology*, Allyn and Bacon, Boston, 1966.
[5] J. L. Kelley, *General Topology*, Van Nostrand Reinhold Co., New York, 1955.

MSMTCS 204	Ordinary Differential Equations	4 Credits (48 hours)

Course Objectives

1. **Study Linear Differential Equations:** Understand linear dependence, the Wronskian, and methods for solving linear differential equations, including variations of parameters and equations with constant coefficients.
2. **Explore Power Series Solutions:** Analyze second-order linear equations with ordinary and singular points, including Legendre and Bessel equations.
3. **Examine Systems of Linear Differential Equations:** Learn to solve systems of first-order equations, apply the fundamental matrix, and address non-homogeneous systems and systems with periodic coefficients.
4. **Understand Existence and Uniqueness:** Investigate the existence and uniqueness of solutions for differential equations, including methods of approximation and Picard's theorem.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Apply methods like the Wronskian, variation of parameters, and handle equations with constant coefficients.
2. Solve second-order linear equations using power series, and work with Legendre and Bessel equations.
3. Solve first-order linear systems, apply the fundamental matrix, and manage systems with periodic coefficients.
4. Use Picard's theorem and methods of successive approximation to analyze solutions and address non-uniqueness.
5. Apply techniques to determine the existence, uniqueness, and continuation of solutions to differential equations.

Contents

Unit I - Linear Differential Equations of Higher Order:

Linear dependence and the Wronskian, Basic theory for linear equations, Method of variation of parameters, Reduction of n^{th} order linear homogeneous equation, Homogeneous and non-homogeneous equations with constant coefficients.

(12 Hours)

Unit II - Solutions in Power Series:

Second order linear equations with ordinary points, Legendre equation and Legendre polynomials, Second order equations with regular singular points, Bessel equation. (18 Hours)

Unit III - Systems of Linear Differential Equations:

Systems of first order equations, Existence and uniqueness theorem. The fundamental matrix, Non-homogeneous linear systems, Linear systems with periodic coefficients. (12 Hours)

Unit IV - Existence and Uniqueness of solutions :

Equations of the form $x' = (t, x)$, Method of successive approximation, Lipschitz condition, Picard's theorem, Non uniqueness of solutions, Continuation of solutions. (6 Hours)

References

[1] S. G. Deo and V. Raghavendra, *Ordinary Differential Equations and Stability Theory*, Tata McGraw Hill, 1980.

[2] A. Coddington, *An Introduction to Ordinary Differential Equations*, Prentice Hall of India, 2013.

[3] A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Krieger, 1984.

[4] M. W. Hirsh and S. Smale, *Differential Equations, Dynamical Systems and Linear Algebra*, Academic Press, New York, 1974. 5. V. I. Arnold, Ordinary Differential Equations, MIT Press, Cambridge, 1981.

[5] Shepley L. Ross, *Differential Equations*, Wiley, 2004.

MSMTCS 205	**Graph Theory**	**4 Credits (48 hours)**

Course Objectives

1. **Understand Fundamental Graph Theory Concepts**: Gain a thorough understanding of the basic concepts in graph theory, including subgraphs, vertex degrees, paths, connectedness, and various operations and products on graphs.
2. **Explore Connectivity in Graphs**: Develop knowledge of connectivity in graphs, including vertex and edge cuts, blocks, and important theorems such as Menger's theorem.
3. **Analyze Trees and Special Graphs**: Study the properties and characterization of trees, understand the concepts of centers and centroids, and learn methods for counting spanning trees. Explore the properties of Eulerian and Hamiltonian graphs.
4. **Learn Graph Colorability**: Understand the principles of graph colorability, including the chromatic number, and theorems related to coloring such as the Five Color Theorem and the chromatic polynomial.

Course Outcomes

1. Ability to define and work with subgraphs, calculate degrees of vertices, identify paths and connected components, and apply various operations and products on graphs.
2. Proficiency in analyzing graph connectivity, including identifying and utilizing vertex and edge cuts, understanding the concept of blocks, and applying Menger's theorem to solve problems.
3. Understanding the definition and characterization of trees, determining centers and centroids, counting spanning trees, and analyzing the properties of Eulerian and Hamiltonian graphs.

4. Ability to determine the chromatic number of graphs, apply the Five Color Theorem, and compute the chromatic polynomial, as well as understanding the implications of graph colorability in various contexts.
5. Enhanced problem-solving skills in graph theory, enabling the application of theoretical concepts to practical problems and the development of strategies for addressing complex graph-related challenges.

Contents

Unit I - **Basic Properties of Graphs**

Introduction, Basic concepts, subgraphs, Degrees of Vertices, Paths and connectedness, The problem of Ramsey, Extremal graphs, Intersection graphs, Operations on graphs, Graph products.

(12 Hours)

Unit II

Connectivity, Vertex cuts and Edge cuts, Connectivity and Edge connectivity, Blocks, Menger;s theorem,.

(12 Hours)

Unit III

Trees - Definition, Characterization, and Simple properties, Centers and centroids, Counting the number of Spanning trees.

Eulerian and Hamiltonian graphs - Eulerian graphs, Hamiltonian graphs and its properties,

(12 Hours)

Unit IV:

Colorability: The chromatic number, The Five Color Theorem, The chromatic polynomial.

Planar Graphs: Planar and non-planar graphs, Euler formula and its consequences, Dual of a plane graph. (12 Hours)

References:

1. R.Balakrishnan and K.Ranganathan – A textbook of Graph Theory, Springer-Verlag, 2000.
2. F. Harary – Graph Theory, Addison-Wesley Series in Mathematics, 1969.
3. Narsingh Deo – Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
4. Bela Bollabas – Modern Graph theory, Springer, 1998.
5. Douglass B. West – Introduction to Graph Theory, Prentice Hall of India, New Delhi, 1996.
6. O. Ore – Theory of Graphs, American Mathematical Society, Providence, Rhode Island, 1967.

MSMTPS 201	**Practical - II**	**2 Credits (2 hours lab /week)**

Mathematics Practicals using Python Programming Language

Course Objectives

1. **Visualize Mathematical Functions**: Develop programs to plot labeled graphs of elementary functions, plane curves, space curves, and surfaces using Python.
2. **Manipulate Matrices**: Write Python programs to compute matrix operations such as transpose, trace, determinant, norm, and perform matrix addition, subtraction, multiplication, and inversion.

3. **Solve Linear Systems**: Implement methods to check the consistency of linear systems, and solve systems using matrix inversion, Cramer's rule, Gauss Elimination, and Gauss-Jordan methods.
4. **Apply Numerical Methods**: Use iterative methods like Jacobi and Gauss-Seidel, and numerical techniques like the Power Method to find eigenvalues and eigenvectors of matrices.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Create clear, labeled graphs of elementary functions, plane curves, space curves, and surfaces using Python libraries.
2. Calculate matrix transpose, trace, determinant, norm, and perform matrix arithmetic and inversion using Python.
3. Check the consistency of linear systems and find solutions using methods like matrix inversion, Cramer's rule, Gauss Elimination, and Gauss-Jordan.
4. Solve systems of equations using Jacobi and Gauss-Seidel iterative methods.
5. Use the Power Method to numerically determine the largest/smallest eigenvalue and corresponding eigenvector of a matrix.

List of Programs

1) Program to plot a neat labeled graph of elementary functions on the same plane.
2) Program to obtain the graph of plane curves - cycloid and astroid in separate figure on a single run.
3) Program to obtain a neat labeled graph of space curves - elliptical helix and circular helix in separate figure on a single run.
4) Program to obtain a neat labeled graph of surfaces - elliptic paraboloid and hyperbolic paraboloid in separate figure on a single run.
5) Program to find the Transpose, Trace, Determinant and Norm of a matrix.
6) Program to find sum, difference and product and inverse (if exists) of matrices.
7) Program to check whether the given system of linear equations are consistent.
8) Program to find solution to a system of linear equations by matrix inversion method (check for all conditions on input matrix).
9) Program to find solution to a system of linear equations by Cramer's rule (check for all conditions on input matrix).
10) Program to solve a system of equations using Gauss Elimination Method and Gauss Jordan Method.
11) Program to find the solution of a system of equations using Jacobi Iterative Method/Gauss Seidal Method.
12) Program to find the numerically largest/smallest eigenvalue and corresponding eigenvector of a matrix by using Power Method.

Semester wise distribution of credits for M.Sc. Mathematics Programme

SEM	Theory(HC[a])		Theory (SC[b])		Open Elective		Lab Credits (SC[b])	Project Credits (HC[a])	Total Credits
	No. of Courses	Credits	No. of Courses	Credits	No. of Courses	Credits			
I	3	4	2	4	-	-	2	-	22
II	3	4	2	4	1	3	2	-	22+3
III	3	4	2	4	1	3	2	-	22+3
IV	2	4	2	4	-	-	-	4	20
Total		44		32	-	6	6	4	86+6

HC[a] - Hard core, SC[b] - Soft core, Not included for CGPA
Total Hard Core Credits is 44+4=48 (55:81%) and total Soft Core Credits is 32+6=38 (44:19%).